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ABSTRACT

This paper considers applications of decision theory to the problem of instructional decision-making in computer-based adaptive instructional systems, using the Minnesota Adaptive Instructional System (MAIS) as an example. The first section indicates how the problem of selecting the appropriate amount of instruction in MAIS can be situated within the general framework of empirical Bayesian decision theory. The linear loss model and the classical test model are discussed in this context. The second section describes six characteristics essential in effective computerized adaptive instructional systems: (1) initial diagnosis and prescription; (2) sequential character of the instructional decision-making process; (3) appropriate amount of instruction for each student; (4) sequence of instruction; (5) instructional time control; and (6) advisement of learning need. It is shown that all but the sequence of instruction could be improved in MAIS with the extensions proposed. Several new lines of research arising from the application of psychometric theory to the decision component in MAIS are reviewed. Thirty-six references and a list of 28 recent research reports of the University of Twente (The Netherlands) Division of Educational Measurement and Data Analysis are included. (MES)

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# APPLICATIONS OF DECISION THEORY TO COMPUTER-BASED ADAPTIVE INSTRUCTIONAL SYSTEMS

**Research Report 88-18**

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# Applications of Decision Theory to Computer-Based Adaptive Instructional Systems

Hans J. Vos

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## Abstract

The purpose of this paper is to consider some applications of decision theory to the problem of instructional decision making in computer-based adaptive instructional systems. In particular, the Minnesota Adaptive Instructional System (MAIS) will be discussed. It will first be indicated how the problem of selecting the appropriate amount of instruction in MAIS can be situated within the general framework of (empirical) Bayesian decision theory. Subsequently, it will be shown how five out of six characteristics identified as essential in an effective computerized adaptive instructional system can be improved by extending MAIS in two directions (viz. adopting a linear loss model and the classical test model).

## Introduction

Computer-based adaptive instructional systems were been studied recently by Atkinson (1976), Ferguson (1970), Hambleton (1974), Hansen, Ross and Rakow (1977), Holland (1977), Park (1982) and Tennyson and Breuer (1984). Although different authors have defined the term "adaptive instruction" in a different way, most agree that it denotes the use of strategies to adapt instructional treatments to the changing nature of student abilities and characteristics during the learning process (Tennyson & Park, 1984).

Four empirically based adaptive instructional modes have been reviewed by Tennyson and Park (1984). The four models are Atkinson's mathematical models, Ross's trajectory model, Ferguson's testing and branching model, and Tennyson's Minnesota Adaptive Instructional System (MAIS). These four models vary in degree to which they use six characteristics identified as essential in an effective adaptive instructional system. The authors conclude that MAIS provides for a complete adaptive instructional model, because all six defined characteristics of adaptive instruction are integrated into this model.

The purpose of this paper is to generalize and extend the application of Bayesian decision theory in MAIS. First, it will be indicated how this model can be situated within the general framework of (empirical) Bayesian decision theory, and what implicit assumptions have to be made in doing so. Subsequently, it will be shown how five out of the

six defined characteristics of computerized adaptive instruction in the MAIS model can be improved by applying other results from decision theory to this model. A linear loss function is proposed to replace the threshold loss function assumed in MAIS. Moreover, optimal sequential decision rules will be derived using Kelley's regression line of classical test theory as the psychometric model instead of the binomial model assumed in MAIS. The paper concludes with a discussion of some new lines of research arising from the application of decision theory to the MAIS model.

## Adapting the Amount of Instruction

Initial work on MAIS began as an attempt to design an adaptive instructional strategy for concept-learning (Tennyson, 1975). According to Merrill and Tennyson (1977), concept-learning can be conceived as a two-stage process of formation of conceptual knowledge and development of procedural knowledge (for a complete review of the theory of concept-learning, see Tennyson and Cocciarella, 1986).

In MAIS, eight basic instructional design variables directly related to specific learning processes are distinguished. In order to adapt instruction to individual learner differences (aptitudes, prior knowledge) and learning needs (amount and sequence of instruction), these variables are controlled by an intelligent tutor system (Tennyson, Christensen & Park, 1984). The authors consider MAIS as an

intelligent system, because it exhibits some machine intelligence, as demonstrated by its ability to improve decision making over the history of the system as a function of accumulated information about previous students. In the literature, successful research projects on MAIS have been reported (e.g., Park & Tennyson, 1980; Tennyson, Tennyson & Rothen, 1980). Three out of the eight instructional design variables are directly managed by a computer-based decision strategy, namely amount of instruction, instructional time control, and advisement on learning need. These three variables also belong to the six characteristics of computerized adaptive instruction. In MAIS, selecting the appropriate amount of instruction can be interpreted as determining the optimal number of interrogatory examples (question form).

The derivation of an optimal strategy with respect to the number of interrogatory examples in a concept-learning lesson requires an instructional problem be stated in a form amenable to decision-theoretic analysis. In the Bayesian view of decision making, there are two basic elements to any decision problem: a loss function describing the loss $l(a_i,\pi)$ incurred when action $a_i$ is taken for the student whose true level of functioning is $\pi$ $(0 \leq \pi \leq 1)$, and a probability function or psychometric model, $f(x|\pi)$, relating observed test scores $x$ to student's true level of functioning.

These basic elements have been related to decision problems in educational testing by many authors, particularly in the context of computer-based adaptive instructional

systems (e.g., Atkinson, 1976; Swaminathan, Hambleton & Algina, 1975; van der Linden, 1981a). As the use of the decision component in MAIS refers to sequential mastery testing, we shall discuss here only the application of the basic elements to this problem.

It is assumed that, due to measurement and sampling errors, the true level of functioning $\pi$ is unknown. All that is known is the student's observed test score X from a small sample of n interrogatory examples (x = 0,1, . . . . . ,n). Furthermore, the following two actions are available to the decision-maker: advance a student ($a_1$) to the next concept if his/her test score X = x exceeds a certain cut-off score c on the observed test score scale X, and retain ($a_0$) him/her otherwise. Students with test score X below the cut-off score c are provided with additional expository examples (statement form). A new interrogatory example is then generated. This procedure is applied sequentially until either mastery is attained or the pool of test items is exhausted. Now, the sequential mastery decision problem can be stated as choosing a value of c that, given the value of the criterion level $\pi_0$, is optimal in some sense. The criterion level $\pi_0 \in [0,1]$ – the minimum degree of mastery required – is set in advance by the decision-maker.

Generally speaking, a loss function specifies the total costs of all possible decision outcomes. These costs concern all relevant psychological, social, and economic consequences which the decision brings along. An example of economic consequences is extra computer time associated with

presenting additional instructional materials. In Tennyson's approach, the loss function is supposed to be a threshold function. The implicit choice of this function implies that the "seriousness" of all possible consequences of the two available actions can be summarized by four constants, one for each of the four possible decision outcomes (see Table 1).

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Insert Table 1 about here

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For convenience, and without loss of generality (e.g., Davis, Hickman & Novick, 1973), it is assumed in Table 1 that no losses occur for correct decisions, and, that therefore, the losses associated with correct advance and retain decisions ($l_{11}$ and $l_{00}$, respectively) can be set equal to zero.

In the decision component of MAIS, a loss ratio R must be specified. R refers to the relative losses associated with advancing a learner whose true level of functioning is below $\pi_0$ and retaining one whose true level exceeds $\pi_0$. From Table 1 it can be seen that the loss ratio R equals $l_{10}/l_{01}$ for all values of $\pi$.

Finally, it is assumed that the psychometric model in MAIS relating observed test scores X to the true level of functioning $\pi$ can be represented by the binomial model,

(1) $$f(x|\pi) = \binom{n}{x} \pi^x (1-\pi)^{n-x}.$$

Within a Bayesian decision-theoretic framework a sequential mastery decision problem is solved by minimizing the "Bayes risk", which is minimal if for each value x of X an action with smallest posterior expected loss is chosen. The posterior expected loss is the expected loss taken with respect to the posterior distribution of $\pi$.

It can be seen from the loss table that a decision rule minimizing posterior expected loss is to advance a student whose test score x is such that

(2) $$l_{10}\text{Prob}(\pi < \pi_0|x,n) \leq l_{01}\text{Prob}(\pi \geq \pi_0|x,n),$$

and to retain him/her otherwise. Since $l_{01} > 0$, this is equivalent to advancing a student if

(3) $$\text{Prob}(\pi \leq \pi_0|x,n) \leq 1/(1+R),$$

and retaining him/her otherwise. $\text{Prob}(\pi \leq \pi_0|x,n)$ denotes the probability of the student's true level of functioning smaller or equal to $\pi_0$ given a test score x on a test of length n. In fact, this probability is given by the cumulative posterior distribution of $\pi$. In MAIS this quantity is called the "beta value" or "operating level" (Tennyson, Christensen & Park, 1984).

It should be noted that, as can be seen from the decision rule, the decision-maker need not specify the values $l_{10}$ and $l_{01}$ completely. He needs only assess their ratio $l_{10}/l_{01}$. For assessing loss functions, most texts on decision theory propose lottery methods (see, for example, Luce & Raiffa, 1957, chap. 2; for a recent modification, see Novick & Lindley, 1979). But in principle any psychological scaling method can be used. Although helpful techniques are available, this does not mean that, for example, in programs of individualized instruction, assessment of losses is always a simple matter. In the next section, we shall consider one method that works in decision problems with a finite number of outcomes such as the sequential mastery decision problem.

In order to initiate the decision component in MAIS, three kinds of parameters must be specified in advance (Rothen & Tennyson, 1984). Beside the parameters $\pi_0$ and R, a probability distribution representing the prior knowledge about $\pi$ must be available. In MAIS, a beta distribution, $B(\alpha,\beta)$, is used as a prior distribution, and a pretest score together with information about other students is used to specify its parameter values.

Keats and Lord (1962) have shown that simple moment estimators for $\alpha$ and $\beta$ can be derived that are based on the mean, the standard deviation, and the KR-21 reliability coefficient of the test scores from the previous students. Let the KR-21 reliability be defined as

$$(4) \qquad r_{21} = \frac{n}{n-1}\left[1 - \frac{\mu_X(n-\mu_X)}{n\sigma^2_X}\right].$$

where $u_X$ and $\sigma^2_X$ denote the mean and the variance of the pretest scores, respectively. Then the estimates $\hat{\alpha}$ and $\hat{\beta}$ of $\alpha$ and $\beta$, respectively, are given as

$$(5) \qquad \hat{\alpha} = (-1 + 1/r_{21})\mu_X$$
$$\hat{\beta} = -\hat{\alpha} + n/r_{21} - n.$$

It follows that the posterior distribution of $\pi$ is easily obtained. From an application of Bayes' theorem, the posterior distribution will again be a member of the beta family (the conjugacy property). In fact, if the prior distribution is $B(\alpha,\beta)$ and the student's test score is $x$ from a test of length $n$, then the posterior distribution is $B(x+\alpha, n-x+\beta)$. The beta distribution has been extensively tabulated (e.g., Pearson, 1930). Normal approximations are also available (Johnson & Kotz, 1970, sect. 2.4.6). In general, if $\pi$ has a beta distribution with parameters $(\alpha,\beta)$ where neither $\alpha$ nor $\beta$ is small (say, not < 10), then this distribution can be approximated by a normal distribution with mean $\alpha/(\alpha+\beta)$ and variance $\alpha\beta/((\alpha+\beta)^2(\alpha+\beta+1))$. These mean and variance are just those of the beta distribution. Tennyson and Christensen (1986) use a non-linear regression approach that fits the best polynomial as an approximation of the beta distribution. Finally, Vijn and Molenaar (1981) have

shown, in the context of robustness regions for dichotomous decisions, that, if we put

$$\tau_0 = \arcsin\sqrt{\pi_0}$$
$$g_N = \arcsin((\alpha+x+3/8)/(\alpha+\beta+n+3/4))^{1/2} \quad (6)$$
$$t_N = (4\alpha+4\beta+4n+2)^{-1/2},$$

expression (3) can be replaced by

$$\tau_0 - g_N \leq -(t_N/1.7)\ln R, \quad (7)$$

from which the optimal cutting score x can easily be evaluated on a computer.

The MAIS decision procedure for adapting the number of interrogatory examples can now be summarized as follows: If the quantity 1/(1+R) exceeds a student's beta value, (s)he is passed to the next instructional unit (i.e., next concept) or final (summative) posttest. However, if his/her beta value is below this quantity, his/her posterior distribution is used as a prior distribution in a next cycle. A new interrogatory example is then generated. The procedure is applied iteratively until either the quantity 1/(1+R) exceeds the beta value, or all interrogatory examples in the pool for the concept have been presented.

In the MAIS decision procedure, it is assumed that the form of the loss structure involved is a threshold function. Therefore, only the loss ratio R has to be assessed empirically. However, a decision-maker's loss structure can

be completely assessed without making any assumptions about the form of the loss function. Only minimal axioms from utility theory have to be assumed. However, as van der Linden (1981a) has pointed out, these techniques do not automatically lead to elegant loss functions and optimal cutting scores. It may be wise, therefore, to use these techniques only for a priorly chosen mathematical form of the loss function. In addition to the threshold loss function, however, more useful functions have been adopted in decision theory. One such function will be considered below.

## The Linear Loss Model

An obvious disadvantage of the threshold loss function is that it assumes constant loss for students to the left or to the right of $\pi_0$, no matter how large their distance from $\pi_0$. For instance, a misclassified true master (see Table 1) with a true level of functioning just above $\pi_0$ gives the same loss as a misclassified true master with a true level far above $\pi_0$. It seems more realistic to suppose that for misclassified true masters the loss is a monotonically decreasing function of the variable $\pi$.

Moreover, as can be seen in Table 1, the threshold loss function shows á "threshold" at the point $\pi = \pi_0$, and this also seems unrealistic in many cases. In the neighbourhood of this point, the losses for correct and incorrect decisions frequently change smoothly rather than abruptly.

In view of this, van der Linden and Mellenbergh (1977) propose a linear loss function:

$$(8) \qquad l(a_i,\pi) = \begin{cases} b_0(\pi-\pi_0)+d_0 & \text{for } i = 0 \text{ (retain)} \\ b_1(\pi_0-\pi)+d_1 & \text{for } i = 1 \text{ (advance)} \end{cases} \qquad b_0,\ b_1 > 0.$$

The above defined function consists of a constant term and a term proportional to the difference between the true level of functioning $\pi$ and the specified criterion level $\pi_0$. The constant amount of loss, $d_j$ (j=0,1), can, for example, represent the costs of testing. The condition $b_0,\ b_1 > 0$ is equivalent to the statement that for actions $a_0$ and $a_1$, utility is a strictly increasing and decreasing function of the variable $\pi$, respectively. The parameters $b_0$, $b_1$, $d_0$, and $d_1$ have to be assessed empirically. Figure 1 displays an example of this function.

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Insert Figure 1 about here

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The linear loss function seems to be a realistic representation of the losses actually incurred in many decision making situations. In a recent study, for example, it was shown by van der Gaag (1987) that many empirical loss structures could be approximated by linear functions.

As the general linear loss function now stands, we need to determine the four constants $b_0$, $b_1$, $d_0$, and $d_1$ before it can be applied. However, if we use the fact that a loss function needs to be determined only up to a positive multiplicative and additive constant (e.g., Luce & Raiffa, 1957), we can reduce the number of unknown constants to two. Thus, since $b_1 > 0$, we may redefine $l(a_i,\pi)$ by making the positive linear transformation $l^*(a_i,\pi) = [l(a_i,\pi)-d_1]/b_1$. And so

$$(9) \qquad l^*(a_i,\pi) = \begin{cases} b^*(\pi-\pi 0)+d^* & \text{for } i = 0 \\ \pi 0 - \pi & \text{for } i = 1, \end{cases}$$

where $b^* = b_0/b_1$ and $d^* = (d_0-d_1)/b_1$.

We turn now to an illustration of one of the most direct methods available for determining the constants $b^*$ and $d^*$. In order to make the method work, the decision-maker must be able to specify two ordered pairs $(\pi_i,\pi_j)$ and $(\pi_i',\pi_j')$ such that

$$(10) \qquad \begin{aligned} &l^*(a_0,\pi_i) = l^*(a_1,\pi_j) \\ &\text{and} \\ &l^*(a_0,\pi_i') = l^*(a_1,\pi_j'). \end{aligned}$$

Solving this system of equations, we find that

$$(11) \qquad b^* = (\pi_j' - \pi_i)/(\pi_i - \pi_i') \text{ and } d^* = \pi_0 - \pi_j - b^*(\pi_i - \pi_0).$$

Choosing the $\pi$–coordinate of the intersection of both loss lines $l^*(a_0,\pi)$ and $l^*(a_1,\pi)$ as one of the ordered pairs, it follows that

$$(12) \qquad \begin{aligned} b^* &= (\pi_j-\pi_p)/(\pi_p-\pi_i) \\ d^* &= (\pi_0-\pi_p)(1+b^*), \end{aligned}$$

where $\pi_p$ denotes the point of intersection.

The decision rule that minimizes the posterior expected loss in the case of a linear loss function is to advance a student with test score x for which

$$(13) \qquad E[(\pi_0-\pi)|x,n] \leq E[b^*(\pi-\pi_0)+d^*|x,n]$$

and to retain him/her otherwise. Since $(1+b^*) > 0$, this is equivalent to advancing a student if

$$(14) \qquad E[\pi|x,n] \geq \pi_0 - d^*/(1+b^*),$$

and retaining him/her otherwise. In other words, with linear loss, the action taken depends only upon the expectation of the posterior distribution of $\pi$, other attributes of the distribution are irrelevant for decision purposes.

Using the fact that the expectation of a beta distribution $B(\alpha,\beta)$ is equal to $\alpha/(\alpha+\beta)$, and, thus, the posterior expectation equals $(\alpha+x)/(\alpha+x+\beta+n-x)$, it follows

that a student is advanced if his/her test score **x** is such that

$$(15) \qquad x \geq [\alpha+\beta+n][\pi_0-d^*/(1+b^*)] - \alpha,$$

and retained otherwise.

Putting $l^*(a_0,\pi)$ and $l^*(a_1,\pi)$ equal to each other, it appears that the $\pi$–coordinate of the intersection of both loss lines from Formula (9), $\pi_p$, is equal to $\pi_0 - d^*/(1+b^*)$. Therefore, the decision rule can be viewed as advancing a student if his/her expectation of the posterior distribution of $\pi$ is to the right of the intersection point, and retaining him/her otherwise. Note that with linear loss, only the point of intersection, $\pi_p$, of the two loss lines for retain and advance are needed, and, thus, the intercept and slope of Formula (9) does not have to be estimated. Hence, expression 15 is equivalent to

$$(16) \qquad x \geq [\alpha+\beta+n]\pi_p - \alpha.$$

When $d^* = 0$, that is, $d_0 = d_1$, both loss lines intersect at $\pi = \pi_0$ and an interesting case arises. Then, all loss function parameters vanish from the decision rule and, thus, it takes the form of advancing a student if

$$(17) \qquad E[\pi|x,n] \geq \pi_0.$$

and retaining him/her otherwise. In other words, if the amounts of constant loss, $d_j$ ($j = 0,1$), for both decisions are equal, or if there are no constant losses at all (i.e., no costs of testing are involved), then there is no need to assess the parameters $d^*$ and $b^*$ in adapting the number of interrogatory examples. In that case, the decision rule can even be simplified to advance a student if his/her expectation of the posterior distribution of $\pi$ is greater than or equal to the specified criterion level $\pi_0$, and to retain him/her otherwise.

In MAIS, it is assumed that the form of the psychometric model relating observed test scores to student's true level of functioning can be represented by the binomial model (Equation 1). In the next section, another psychometric model used in criterion-referenced testing will be considered.

## Classical Test Model

The expectation of the posterior distribution, $E[\pi|x,n]$, represents the regression of $\pi$ on $x$. A possible regression function is the linear regression function of classical test theory (Lord & Novick, 1968):

$$(18) \qquad E[\pi|x,n] = \rho_{XX'}\cdot\frac{x}{n} + (1-\rho_{XX'})\frac{\mu_X}{n},$$

$\mu_X$ and $\rho_{XX'}$ being the mean and reliability coefficient of X.

Equation 18 is known as Kelley's regression line. This is an interesting equation in that it expresses the estimate of the true level of functioning as a weighted sum of two separate estimates – one based upon the student's observed score, x, and, the other based upon the mean, $\mu_X$, of the group to which s(he) belongs. If the test is highly reliable, much weight is given to the test score and little to the group mean, and vice versa.

Substituting Equation 18 into expression 14, and solving for x gives the following optimal sequential decision rule

$$(19) \qquad x \geq \mu_X + \frac{n\{\pi_0 - d^*/(1+b^*)\} - \mu_X}{\rho_{XX'}}.$$

If the amounts of constant loss for both decisions are equal, i.e. $d^* = 0$, or if there are no constant losses at all, expression 19 will take the rather simple form

$$(20) \qquad x \geq \mu_X + \frac{n\pi_0 - \mu_X}{\rho_{XX'}},$$

or

$$(21) \qquad x \geq \frac{\mu_X(\rho_{XX'}-1) + n\pi_0}{\rho_{XX'}}.$$

Since $0 \leq \rho_{XX'} \leq 1$, and, thus $-1 \leq \rho_{XX'}-1 \leq 0$, it follows from expression (21) that $\mu_X$ and the optimal sequential cutting score, x, are related negatively. The higher the average performance, the lower the optimal

sequential cutting score. Hard-working students are rewarded by low cutting scores, while less hard-working students will just be penalized and confronted with high cutting scores. This is the opposite of what happens when norm-referenced standards are used (van der Linden & Mellenbergh, 1987).

It should be stressed that, as can be seen from (20), the optimal sequential cutting score, i.e. the number of interrogatory examples to be administered to the student, depends upon $\mu_X$ and $\rho_{XX'}$, the decision component in MAIS allows for an updating after each response to an interrogatory example. This explains why, though the decisions for determining the optimal number of interrrogatory examples are made with respect to an individual student, the rules for the decisons are based on data from all students taught by the system in the past and, in doing so, are improved continuously. In other words, a computer-based adaptive instructional system can be designed in this way, i.e., a system of rules improving itself over the history of the system as a result of systematically using accumulated data from previous students. The parameters of the model, $\mu_X$ and $\rho_{XX'}$, are updated each time a next student has finished his/her dialogue with the system. Kimball (1971) entitles such systems as "self-optimizing".

## Characteristics of Adaptive Instruction in the Extended MAIS

In this section, the six defined characteristics of computerized adaptive instruction according to Tennyson and Park will be discussed for MAIS with the extensions proposed in the present paper.

The first characteristic is initial diagnosis and prescription. MAIS uses this characteristic by administering a pretest (pretask data) to all students from which an initial prior estimate of a student's ability results. The prior distribution is supposed to be a beta distribution, $B(\alpha,\beta)$. Estimates of the parameters $\alpha$ and $\beta$ are given by Equation 5. As an aside, it may be noted that if administering a pretest is not possible for any reason, the prior distribution of a student can be characterized by a uniform distribution on the interval from zero to one. In that case, the parameters of the beta prior should be specified as $\alpha = \beta = 1$. Also, the use of Kelley's regression line (Equation 18) as an estimate of the true score can be considered as an informal way to take account of prior knowledge.

The second characteristic concerns the sequential character of the instructional decision-making process. MAIS uses this characteristic when updating the beta value after each interrogatory example (on-task data) and comparing this value to the quantity $1/(1+R)$ (Expression 3). Using a linear loss function, the posterior expectation of the true level of

functioning should be compared with the point of intersection of the two loss lines for the retain and advance decisions (Expression 12). Moreover, when the amounts of constant loss are equal or, if there are no constant losses at all, the sequential decision rule can even be simplified to comparing the posterior expectation with the criterion level, $\pi_0$, set in advance (Expression 17). Finally, it has been indicated in the previous section that the sequential rules for the instructional decision making process can even be based on data from all students taught by the system in the past, if we adopt the classical test model as the psychometric model actually involved (Expression 19).

The third characteristic refers to the appropriate amount of instruction each student receives to achieve the defined instructional objectives. As mentioned before, in MAIS this is done by determining the optimal number of interrogatory examples. Besides the threshold loss function assumed in MAIS, the optimal number of interrogatory examples has been determined in case a linear loss function as well as Kelley's regression line of classical test theory is adopted.

The fourth characteristic pertains to the sequence of instruction. In MAIS this characteristic follows closely a cognitive theory of concept and rule learning. Since this characteristic is not controlled by the decision component, we refer for further discussion to Tennyson, Youngsters and Suebsonthi (1983).

The fifth characteristic is instructional time control. Control of the instructional time is associated directly with

the Bayesian beta values established for the necessary amount of instruction. The proposed extensions of MAIS described in this paper, namely the linear loss model and the classical test model, can be applied to control the instructional time in a possibly even better way by using the improved optimal sequential decision rules.

Finally, the last characteristic concerns the advisement of learning need. This advisement provides the necessary information by which students can make adequate self-assessments and learning needs judgment. As noted before, this characteristic is one of the eight instructional design variables within MAIS and is also directly managed by the decision component. Analogous to the previous characteristic, it might be expected that by using the proposed linear loss model and the classical test model, this characteristic can be used in such a way that MAIS can even be made more effective.

In summary, five out of the six characteristics identified as essential in an effective computerized adaptive instructional system can be improved by applying the extensions discussed in this paper.

## New Lines of Research

There are a few new lines of research arising from the application of psychometric theory to the decision component in MAIS. The first is the extension of the work of Tennyson

and his associates to situations where guessing and carelessness are incorporated. Morgan (1979) has developed a model with corrections for guessing and carelessness within a Bayesian decision-theoretic framework. The results of a computer simulation of the model indicate that guessing and carelessness may markedly affect the determination of cutting scores, and hence the accuracy of decisions about mastery.

The second line is research into other prior distributions for $\pi$ (for example, the standard normal distribution) than the beta distribution assumed in MAIS. It might also be assumed that no prior distribution about $\pi$ is available, because specifying such a distribution is too difficult a job to accomplish. In these circumstances, the minimax procedure may be an appropriate framework (e.g., Huynh, 1980; van der Linden, 1981b) which requires no prior distribution regarding the true level of functioning. In this case, the optimum cutting score is obtained by minimizing the maximum risk which would incurred by misclassifications. As an aside, it might be noted that a minimax rule can be conceived as a rule that minimizes the "Bayes risk" as well, but under the restriction that the prior is the least favorable of the class of priors (e.g., Ferguson, 1967, sect.1.6.).

The third line of research concerns still other loss structures than the threshold and linear loss function, which represent possibly even more realistic forms that loss structures might take in applications to instructional decision making in computer-based adaptive instructional

systems. For example, the normal ogive function (Novick & Lindley, 1978) which takes loss to be a nonlinearly function of the true level of functioning $\pi$. This loss function does not only have realistic properties but also can be combined nicely with a normal model for the test data.

Finally, an interesting new line of research seems to be an extension of the actions available to the decision-maker. In MAIS, two actions were available to the decision-maker, namely advancing ($a_1$) or retaining ($a_0$) a student. However, it might also be assumed that there are three (or any finite number) of actions open to the decision-maker. For example, in the three-action problem the student may provided with additional instructional materials both of the present and the previous concept ($a_2$); (s)he may provided only with additional instructional materials of the present concept ($a_0$); or, (s)he may advance to the next concept ($a_1$).

We might think of this problem in terms of specifying two cutting scores $c_0$ and $c_1$ on the observed test score scale X, where $c_0 < c_1$. Then for observed test score $x < c_0$, action $a_2$ will be taken; for $c_0 < x < c_1$, action $a_0$ will be taken; and, for $x > c_1$, action $a_1$ will be taken.

Davis, Hickman and Novick (1973) have given a solution to the three-action problem using a natural extension of the threshold loss function. Although the notation becomes more complex and the computation a bit more tedious, there are no fundamentally new ideas in the multiple-action problem.

## Some Summary Remarks

In this paper it was indicated how the MAIS decision procedure could be formalized within a Bayesian decision-theoretic framework. In fact, it turned out that this decision can be considered as a sequential mastery decision.

Moreover, it was argued that in many situations the assumed threshold loss function in MAIS is an unrealistic representation of the loss actually incurred. Instead, a linear loss function was proposed to meet the objections to threshold loss.

Further, Kelley's regression line of classical test theory was proposed as the psychometric model relating observed test scores to the true level of functioning. Using this psychometric model instead of the binomial model assumed in MAIS, computer-based adaptive instructional systems can be designed in which the determination of the optimal number of interrogatory examples for an individual student is based on data from all students taught by the system in the past.

With these two extensions of the MAIS model, it might be expected that five out of the six characteristics of computerized adaptive instruction can even be improved in this model.

Whether or not the proposed linear loss model and the classical test model are, however, real improvements of the present decision component in MAIS (in terms of student performance on posttests, learning time, and amount of

instruction) must be decided on the basis of empirical experiments.

## References

Atkinson, R.C. (1976). Adaptive instructional systems: Some attempts to optimize the learning process. In D. Klahr (Ed.), Cognition and instruction. New York: John Wiley & Sons.

Davis, C.E., Hickman, J., & Novick, M.R. (1973). A primer on decision analysis for individually prescribed instruction (ACT Technical Bulletin No. 17). Iowa City, IA: The American College Testing Program.

DeGroot, M.H. (1970). Optimal statistical decisions. New York: McGraw-Hill.

Ferguson, R.A. (1970). A model for computer-assisted criterion-referenced measurement. Education, 91, 25-31.

Ferguson, T.S. (1967). Mathematical statistics: A decision theoretic approach. New York: Academic Press.

Hambleton, R.K. (1974). Testing and decision-making procedures for selected individualized programs. Review of Educational Research, 44, 371-400.

Hansen, D.N., Ross, S.M., & Rakow, E. (1977). Adaptive models for computer-based training systems (Annual report to Naval Personnel Research and Development Center). Memphis, TN: Memphis State University.

Holland, J.G. (1977). Variables in adaptive decisions in individualized instruction. Educational Psychologist, 12, 146-161.

Huynh, H. (1980). A nonrandomized minimax solution for passing scores in the binomial error model. Psychometrika, 45, 167-182.

Johnson, N.L., & Kotz, S. (1970). Distributions in statistics: Continuous univariate distributions - 2. Boston: Houghton Mifflin.

Keats, J.A., & Lord, F.M. (1962). A theoretical distribution for mental test scores. Psychometrika, 27, 59-72.

Kimball, R.B. (1973). Self-optimizing computer-assisted tutoring: theory and practice. Unpublished Thesis, Mathematical Studies in Social Science, Stanford University.

Lindgren, B.W. (1976). Statistical theory (3rd ed.). New York: Macmillan.

Lord, F.M., & Novick, M.R. (1968). Statistical theories of mental test scores. Reading, M.A.: Addison Wesley.

Luce, R.D., & Raiffa, H. (1957). Games and decisions. New York: John Wiley & Sons.

Merrill, M.D., & Tennyson, R.D. (1977). Concept teaching: An instructional design guide. Englewood Cliffs, New York: Educational Technology.

Morgan, G. (1979). A criterion-referenced measurement model with corrections for guessing and carelessness (Occasional Paper No. 13). Victoria: The Australian Council for Educational Research Limited.

Novick, M.R., & Jackson, P.H. (1974). Statistical methods for educational and psychological research. New York: McGraw-Hill.

Novick, M.R., & Lindley, D.V. (1979). Fixed-state assessment of utility functions. Journal of the American Statistical Association, 74.

Park, O. (1982). A response-sensitive strategy in computer-based instruction. Journal of Educational Technology Systems, 10, 187-197.

Park, O., & Tennyson, R.D. (1980). Adaptive design strategies for selecting number and presentation order of examples in coordinate concept acquisition. Journal of Educational Psychology, 72, 362-370.

Pearson, K. (1930). Tables for statisticians and biometricians, 2. London: Cambridge University Press.

Swaminathan, H., Hambleton, R.K., & Algina, J. (1975). A Bayesian decision-theoretic procedure for use with criterion-referenced tests. Journal of Educational Measurement, 12, 87-98.

Tennyson, R.D. (1975). Adaptive instructional models for concept acquisition. Educational Technology, 15, 7-15.

Tennyson, R.D., & Breuer, K. (1984). Cognitive-based design guidelines for using video and computer technology in higher education. In O. Zuber-Sherritt (Ed.), Video in higher education. London, England: Kogan Page Ltd., 26-630.

Tennyson, R.D., Christensen, D.L., & Park, S.I. (1984). The Minnesota Adaptive Instructional System: An intelligent CBI system. Journal of Computer-Based Instruction, 11, 2-13.

Tennyson, R.D., & Christensen, D.L. (1986). MAIS: An intelligent learning system. In D. Jonassen (Ed.), Instructional designs for microcomputer courseware. Englewood Cliffs, New York: Lawrence Erlbaum Associates.

Tennyson, R.D., & Cocchiarella, M.J. (1986). An empirically based instructional design theory for teaching concepts. Review of Educational Research, 56.

Tennyson, R.D., & Park, O. (1984). Computer-based adaptive instructional systems: A review of empirically based models. Machine-Mediated Learning, 1, 129-153.

Tennyson, C.L., Tennyson, R.D., & Rothen, W. (1980). Content structure and management strategies as design variables in concept acquisition. Journal of Educational Psychology, 72, 499-505.

Tennyson, R.D., Youngsters, Y., & Suebsonthi, P. (1983). Concept learning by children using instructional presentation forms for prototype formation and classification skill development. Journal of Educational Psychology, 75, 280-291.

van der Gaag, N.L. (1987). Application of empirical linear utility functions to mastery decisions. Paper presented at the European Psychometric Society, Enschede, The Netherlands.

van der Linden, W.J. (1981a). Using aptitude measurements for the optimal assignment of subjects to treatments with and without mastery scores. Psychometrika, 46, 257-274.

van der Linden, W.J. (1981b). Decision models for use with criterion-referenced tests. Applied Psychological Measurement, 4, 469-492.

van der Linden, W.J., & Mellenbergh, G.J. (1977). Optimal cutting scores using a linear loss function. Applied Psychological Measurement, 1, 593-599.

Vijn, P., & Molenaar, I.W. (1981). Robustness regions for dichotomous decisions. Journal of Educational Statistics, 6, 205-235.

## Author's Note

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Table 1

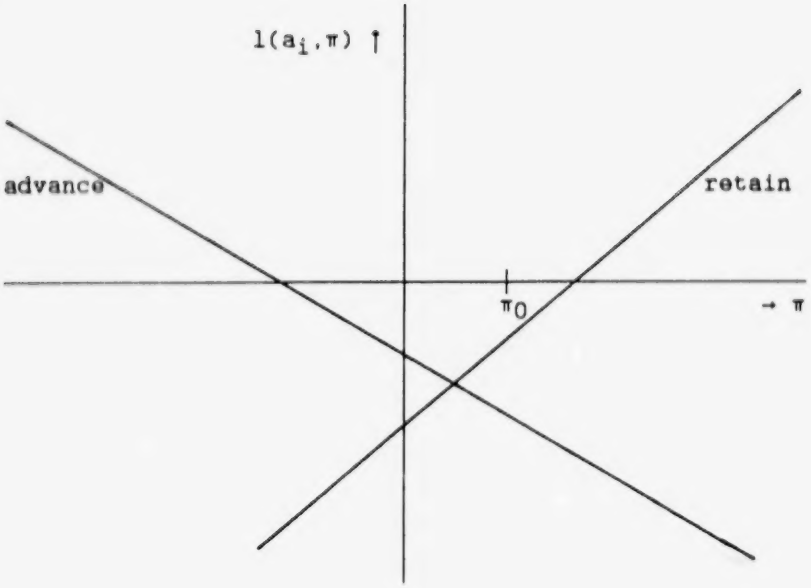
Twofold Table for Threshold Loss Function

| | | True level | |
|---|---|---|---|
| | | $\pi \geq \pi_0$ (true master) | $\pi < \pi_0$ (true nonmaster) |
| Decision | Advance | 0 | $l_{10}$ |
| | Retain | $l_{01}$ | 0 |

Figure Caption

Figure 1. An Example of a Linear Loss Function.

($b_0 \neq b_1$, $d_0 \neq d_1$)


l(a_i, π) ↑
advance
retain
π0
→ π

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